CHERN-SIMONS $D=3, N=6$ SUPERFIELD THEORY

B.M. Zupnik
Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, Moscow Region, 141980, Russia; E-mail: zupnik@theor.jinr.ru

Abstract

We construct the D=3, N=5 harmonic superspace using the SO(5)/U(1)×U(1) harmonics. Three gauge harmonic superfields satisfy the off-shell constraints of the Grassmann and harmonic analyticities. The corresponding component supermultiplet contains the gauge field A_m and an infinite number of bosonic and fermionic fields with the SO(5) vector indices arising from decompositions of gauge superfields in harmonics and Grassmann coordinates. The nonabelian superfield Chern-Simons action is invariant with respect to the N=6 superconformal supersymmetry realized on the N=5 superfields. The component Lagrangian contains the Chern-Simons interaction of A_m and an infinite number of bilinear and trilinear interactions of auxiliary fields. The fermionic and bosonic auxiliary fields from the infinite N=5 multiplet vanish on-shell.

PACS: 11.30. Pb, 11.15. Tk
Keywords: Harmonic superspace; Grassmann and harmonic analyticities; Chern-Simons theory

1 Introduction

Supersymmetric extensions of the three-dimensional Chern-Simons (CS) theory were discussed in refs. [1]-[11]. The N=1 CS theory of the spinor gauge superfield [1, 2] was constructed in the D=3, N=1 superspace with real coordinates x^m, θ^α, where $m = 0, 1, 2$ is the 3D vector index and $\alpha = 1, 2$ is the SL(2,R) spinor index. The N=1 CS action can be interpreted as the superspace integral of the Chern-Simons superform $dA + \frac{2}{3}A^3$ in the framework of our theory of superfield integral forms [3]-[5].

The abelian N=2 CS action was first constructed in the D=3, N=1 superspace [1]. The nonabelian N=2 CS action was considered in the D=3, N=2 superspace in terms of the Hermitian superfield $V(x^m, \theta^\alpha, \bar\theta^\alpha)$ (prepotential) [3, 9, 10], where θ^α and $\bar\theta^\alpha$ are the complex conjugated N=2 spinor coordinates. The corresponding component-field Lagrangian includes the bosonic CS term and the bilinear terms with fermionic and scalar fields without derivatives. The unusual dualized form of the N=2 CS Lagrangian contains the second vector field instead of the scalar field[10].

The D=3, N=3 CS theory was first analyzed by the harmonic-superspace method [6, 7]. Note that the off-shell N=3 and N=4 vector supermultiplets are identical [13]; however, the superfield CS action is invariant with respect to the $N = 3$ supersymmetry only. Nevetheless, the N=3 CS equations of motion are covariant under the 4th supersymmetry. The field-component form of the N=3 CS Lagrangian was studied in [8].

The off-shell D=3, N=6 SYM theory arises by a dimensional reduction of the D=4, N=3 SYM theory in the SU(3)/U(1)×U(1) harmonic superspace [12]. Three basic prepotentials of the D=3, N=6 gauge theory contain an infinite number of auxiliary fields with

the SU(3) indices, and a coupling constant of this model has a dimension 1/2. We do not know how to construct the D=3, N=6 CS theory from these gauge harmonic superfields. Note that the SU(3)/U(1)×U(1) analytic harmonic superspace has the integration measure of dimension 1 in the case D=3.

In this paper, we consider the simple D=3, N=5 superspace which cannot be obtained by a dimensional reduction of the even coordinate from any 4D superspace. The corresponding harmonic superspace using the SO(5)/U(1)×U(1) harmonics is discussed in Section 2. The Grassmann-analytic D=3, N=5 superfields depend on 6 spinor coordinates, so the analytic-superspace integral measure has a zero dimension. It is shown that five harmonic derivatives preserve the Grassmann analyticity.

In Section 3, we consider five basic gauge superfields in the D=3, N=5 analytic superspace and the corresponding gauge group with analytic superfield parameters. To simplify the superfield formalism of the theory, one can introduce additional off-shell harmonic-analyticity constraints for the gauge-group parameters and gauge superfields. These harmonic constraints yield additional reality conditions for components of superfields. In this convenient representation, two gauge superfields vanish, and we use only three basic gauge superfields (prepotentials). The Chern-Simons superfield action can be constructed from these D=3, N=5 gauge superfields. We show that this CS superfield action is invariant with respect to the D=3, N=6 superconformal supersymmetry transformations realized on the N=5 superfields. The superfield gauge equations of motion have only pure gauge solutions, by analogy with the N=1, 2, 3 superfield CS theories.

The field-component structure of our D=3, N=6 Chern-Simons model is analyzed in Section 4. In the abelian case, the basic gauge superfield includes the gauge field A_m and the fermion field ψ_α in the SO(5) invariant sector, and an infinite number of fermionic and bosonic fields with the SO(5) vector or tensor indices. The component Lagrangian contains the Chern-Simons term for A_m and the simple bilinear and trilinear interactions of other fermionic and bosonic fields. The field strength of the gauge field and all other fields vanish on-shell.

The preliminary version of the D=3, N=5 harmonic-superspace gauge theory without the harmonic-analyticity conditions was presented in our talk [14]. This model describes the interaction of the N=5 Chern-Simons multiplet with some unusual matter fields.

2 $D = 3, N = 5$ harmonic superspace

The CB-representation of the D=3, N=5 superspace uses three real even coordinates $x^m \quad (m = 0, 1, 2)$ and five two-component odd coordinates θ^α_a, where $\alpha = 1, 2$ is the spinor index of the group SL(2,R) and $a = 1, 2, \dots 5$ is the vector index of the automorphism group SO(5). We use the real traceless or symmetric representations of the 3D g_m matrices

$$(\gamma_m)^{\alpha\beta} = \varepsilon^{\alpha\rho}(\gamma_m)^\beta_\rho = (\gamma_m)^{\beta\alpha}, \quad (\gamma^m)_{\alpha\beta}(\gamma_m)^{\rho\gamma} = \delta^\rho_\alpha\delta^\gamma_\beta + \delta^\rho_\beta\delta^\gamma_\alpha$$
$$(\gamma_m\gamma_n)^\beta_\alpha = (\gamma_m)^\rho_\alpha(\gamma_n)^\beta_\rho = -(\gamma_m)_{\alpha\rho}(\gamma_n)^{\rho\beta} = -\eta_{mn}\delta^\beta_\alpha + \varepsilon_{mnp}(\gamma^p)^\beta_\alpha. \qquad (2.1)$$

where $\eta_{mn} = \mathrm{diag}(1, -1, -1)$ is the 3D Minkowski metric and ε_{mnp} is the antisymmetric symbol.

One can consider the bispinor representation of the 3D coordinates and derivatives

$$x^{\alpha\beta} = (\gamma_m)^{\alpha\beta}x^m, \quad \partial_{\alpha\beta} = (\gamma^m)_{\alpha\beta}\partial_m. \qquad (2.2)$$

The N=5 CB spinor derivatives are

$$D_{a\alpha} = \partial_{a\alpha} + i\theta^{\beta}_{a}\partial_{\alpha\beta}, \quad \partial_{a\alpha}\theta^{\beta}_{b} = \delta_{ab}\delta^{\beta}_{\alpha}. \tag{2.3}$$

The N=5 supersymmetry transformations are

$$\delta_\epsilon x^m = -i\epsilon^\alpha_a(\gamma^m)_{\alpha\beta}\theta^\beta_a, \quad \delta_\epsilon\theta^\alpha_a = \epsilon^\alpha_a. \tag{2.4}$$

We shall use the SO(5)/U(1)×U(1) vector harmonics defined via the components of the real orthogonal 5×5 matrix

$$U^K_a = \left(U^{(1,1)}_a, U^{(1,-1)}_a, U^{(0,0)}_a, U^{(-1,1)}_a, U^{(-1,-1)}_a\right) \tag{2.5}$$

where a is the SO(5) vector index and the index $K = 1, 2, \ldots 5$ corresponds to given combinations of the U(1)×U(1) charges. These harmonics satisfy the following conditions:

$$\begin{aligned} &U^K_a U^L_a = g^{KL} = g^{LK}, \quad g^{KL}U^K_a U^L_b = \delta_{ab}, \\ &g^{15} = g^{24} = g^{33} = 1, \quad g^{11} = g^{12} = \cdots = g^{45} = g^{55} = 0, \end{aligned} \tag{2.6}$$

where g^{LK} is the antidiagonal symmetric constant metric in the space of charged indices.

Let us introduce the following harmonic derivatives:

$$\partial^{KL} = U^K_b g^{LM}\frac{\partial}{\partial U^M_b} - U^L_b g^{KM}\frac{\partial}{\partial U^M_b} = -\partial^{LK}, \tag{2.7}$$

$$[\partial^{IJ}, \partial^{KL}] = g^{JK}\partial^{IL} + g^{IL}\partial^{JK} - g^{IK}\partial^{JL} - g^{JL}\partial^{IK}, \tag{2.8}$$

which satisfy the commutation relations of the Lie algebra SO(5). We will mainly use the five harmonic derivatives and the corresponding U(1)×U(1) notation

$$\begin{aligned} &\partial^{12} = \partial^{(2,0)} = U^{(1,1)}_b\partial/\partial U^{(-1,1)}_b - U^{(1,-1)}_b\partial/\partial U^{(-1,-1)}_b, \\ &\partial^{13} = \partial^{(1,1)} = U^{(1,1)}_b\partial/\partial U^{(0,0)}_b - U^{(0,0)}_b\partial/\partial U^{(-1,-1)}_b, \\ &\partial^{23} = \partial^{(1,-1)} = U^{(1,-1)}_b\partial/\partial U^{(0,0)}_b - U^{(0,0)}_b\partial/\partial U^{(-1,1)}_b, \\ &\partial^{14} = \partial^{(0,2)} = U^{(1,1)}_b\partial/\partial U^{(1,-1)}_b - U^{(-1,1)}_b\partial/\partial U^{(-1,-1)}_b, \\ &\partial^{25} = \partial^{(0,-2)} = U^{(1,-1)}_b\partial/\partial U^{(1,1)}_b - U^{(-1,-1)}_b\partial/\partial U^{(-1,1)}_b. \end{aligned}$$

The Cartan charges of two U(1) groups are described by the neutral harmonic derivatives

$$\partial^0_1 = \partial^{15} + \partial^{24}, \quad \partial^0_1 U^{(p,q)}_a = p\, U^{(p,q)}_a, \quad \partial^0_2 = \partial^{15} - \partial^{24}, \quad \partial^0_2 U^{(p,q)}_a = q\, U^{(p,q)}_a. \tag{2.9}$$

The harmonic integral has the following simple properties:

$$\int dU = 1, \quad \int dU\, U^{(p,q)}_a U^{(-r,-s)}_b = \tfrac{1}{5}\delta_{ab}\delta_{pr}\delta_{qs}. \tag{2.10}$$

Let us define the harmonic projections of the N=5 Grassmann coordinates

$$\theta^K_\alpha = \theta_{a\alpha}U^K_a = (\theta^{(1,1)}_\alpha, \theta^{(1,-1)}_\alpha, \theta^{(0,0)}_\alpha, \theta^{(-1,1)}_\alpha, \theta^{(-1,-1)}_\alpha). \tag{2.11}$$

Using the harmonic-superspace method one can define the coordinates of the N=5 analytic superspace with only three spinor coordinates

$$\zeta=(x^m_{\scriptscriptstyle A},\theta^{(1,1)}_\alpha,\theta^{(1,-1)}_\alpha,\theta^{(0,0)}_\alpha),\tag{2.12}$$

$$x^m_{\scriptscriptstyle A}=x^m+i\theta^{(1,1)}\gamma^m\theta^{(-1,-1)}+i\theta^{(1,-1)}\gamma^m\theta^{(-1,1)},$$
$$\delta_\epsilon x^m_{\scriptscriptstyle A}=-i\epsilon^{(0,0)}\gamma^m\theta^{(0,0)}-2i\epsilon^{(-1,1)}\gamma^m\theta^{(1,-1)}-2i\epsilon^{(-1,-1)}\gamma^m\theta^{(1,1)},\tag{2.13}$$

where $\epsilon^{K\alpha}=\epsilon^\alpha_aU^K_a$ are the harmonic projections of the supersymmetry parameters.

General superfields in the analytic coordinates depend also on additional spinor coordinates $\theta^{(-1,1)}_\alpha$ and $\theta^{(-1,-1)}_\alpha$. The harmonized partial spinor derivatives are

$$\begin{aligned}&\partial^{(-1,-1)}_\alpha=\partial/\partial\theta^{(1,1)\alpha},\quad \partial^{(-1,1)}_\alpha=\partial/\partial\theta^{(1,-1)\alpha},\quad \partial^{(0,0)}_\alpha=\partial/\partial\theta^{(0,0)\alpha},\\ &\partial^{(1,1)}_\alpha=\partial/\partial\theta^{(-1,-1)\alpha},\quad \partial^{(1,-1)}_\alpha=\partial/\partial\theta^{(-1,1)\alpha}.\end{aligned}\tag{2.14}$$

Ordinary complex conjugation connects harmonics of the opposite charges

$$\overline{U^{(1,1)}_a}=U^{(-1,-1)}_a,\quad \overline{U^{(1,-1)}_a}=U^{(-1,1)}_a,\quad \overline{U^{(0,0)}_a}=U^{(0,0)}_a.\tag{2.15}$$

We use the combined conjugation $\sim$ in the harmonic superspace

$$\begin{aligned}&\widetilde{U^{(p,q)}_a}=U^{(p,-q)}_a,\quad \widetilde{\theta^{(p,q)}_\alpha}=\theta^{(p,-q)}_\alpha,\quad \widetilde{x^m_{\scriptscriptstyle A}}=x^m_{\scriptscriptstyle A},\\ &(\theta^{(p,q)}_\alpha\theta^{(s,r)}_\beta)^\sim=\theta^{(s,-r)}_\beta\theta^{(p,-q)}_\alpha,\quad \widetilde{f(x_{\scriptscriptstyle A})}=\bar f(x_{\scriptscriptstyle A}),\end{aligned}\tag{2.16}$$

where $\bar f$ is the ordinary complex conjugation. The analytic superspace is real with respect to the combined conjugation.

One can define the combined conjugation for the harmonic derivatives of superfields

$$\begin{aligned}&(\partial^{(\pm1,1)}A)^\sim=\partial^{(\pm1,-1)}\tilde A,\quad (\partial^{(\pm1,-1)}A)^\sim=\partial^{(\pm1,1)}\tilde A,\\ &(\partial^{(\pm2,0)}A)^\sim=-\partial^{(\pm2,0)}\tilde A,\quad (\partial^{(0,\pm2)}A)^\sim=\partial^{(0,\mp2)}\tilde A.\end{aligned}\tag{2.17}$$

The analytic-superspace integral measure contains partial spinor derivatives (2.14)

$$d\mu^{(-4,0)}=-\frac{1}{64}dUd^3x_{\scriptscriptstyle A}(\partial^{(-1,-1)})^2(\partial^{(-1,1)})^2(\partial^{(0,0)})^2=dUd^3x_{\scriptscriptstyle A}d^6\theta^{(-4,0)},\tag{2.18}$$

$$\int d^6\theta^{(-4,0)}(\theta^{(1,1)})^2(\theta^{(1,-1)})^2(\theta^{(0,0)})^2=1.$$

It is pure imaginary

$$(d\mu^{(-4,0)})^\sim=-d\mu^{(-4,0)},\quad (d^6\theta^{(-4,0)})^\sim=-d^6\theta^{(-4,0)}.\tag{2.19}$$

The harmonic derivatives of the analytic basis commute with the generators of the N=5 supersymmetry

$$\begin{aligned}&\mathcal{D}^{(1,1)}=\partial^{(1,1)}-i\theta^{(1,1)}_\alpha\theta^{(0,0)}_\beta\partial^{\alpha\beta}-\theta^{(0,0)\alpha}\partial^{(1,1)}_\alpha+\theta^{(1,1)\alpha}\partial^{(0,0)}_\alpha,\\ &\mathcal{D}^{(1,-1)}=\partial^{(1,-1)}-i\theta^{(1,-1)}_\alpha\theta^{(0,0)}_\beta\partial^{\alpha\beta}-\theta^{(0,0)\alpha}\partial^{(1,-1)}_\alpha+\theta^{(1,-1)\alpha}\partial^{(0,0)}_\alpha=-(\mathcal{D}^{(1,1)})^\dagger,\\ &\mathcal{D}^{(2,0)}=[\mathcal{D}^{(1,-1)},\mathcal{D}^{(1,1)}]=\partial^{(2,0)}-2i\theta^{(1,1)}_\alpha\theta^{(1,-1)}_\beta\partial^{\alpha\beta}-\theta^{(1,-1)\alpha}\partial^{(1,1)}_\alpha+\theta^{(1,1)\alpha}\partial^{(1,-1)}_\alpha,\\ &\mathcal{D}^{(0,2)}=\partial^{(0,2)}+\theta^{(1,1)\alpha}\partial^{(-1,1)}_\alpha-\theta^{(-1,1)\alpha}\partial^{(1,1)}_\alpha\\ &\mathcal{D}^{(0,-2)}=-(\mathcal{D}^{(0,2)})^\dagger=\partial^{(-2,0)}+\theta^{(1,-1)\alpha}\partial^{(-1,-1)}_\alpha-\theta^{(-1,-1)\alpha}\partial^{(1,-1)}_\alpha.\end{aligned}$$

Note that harmonic derivatives $\mathcal{D}^{(0,\pm 2)}$ change the second U(1) charge; these operators do not act on x^m_A, in distinction with other harmonic derivatives. It is useful to define the AB-representation of the U(1) charge operators

$$\mathcal{D}^0_1 A^{(p,q)} = p\, A^{(p,q)}, \quad \mathcal{D}^0_2 A^{(p,q)} = q\, A^{(p,q)}, \tag{2.20}$$

where $A^{(p,q)}$ is an arbitrary harmonic superfield in AB.

The spinor derivatives in the analytic basis are

$$\begin{aligned}
&D^{(-1,-1)}_\alpha = \partial^{(-1,-1)}_\alpha + 2i\theta^{(-1,-1)\beta}\partial_{\alpha\beta}, \quad D^{(-1,1)}_\alpha = \partial^{(-1,1)}_\alpha + 2i\theta^{(-1,1)\beta}\partial_{\alpha\beta},\\
&D^{(0,0)}_\alpha = \partial^{(0,0)}_\alpha + i\theta^{(0,0)\beta}\partial_{\alpha\beta}, \quad D^{(1,1)}_\alpha = \partial^{(1,1)}_\alpha, \quad D^{(1,-1)}_\alpha = \partial^{(1,-1)}_\alpha.
\end{aligned} \tag{2.21}$$

The analytic superfields $\Lambda(\zeta, U)$ depend on harmonics and the analytic coordinates, and satisfy the Grassmann analyticity conditions

$$G: \qquad D^{(1,\pm 1)}_\alpha \Lambda = 0. \tag{2.22}$$

The action of the five harmonic derivatives

$$\mathcal{D}^{(1,\pm 1)}, \quad \mathcal{D}^{(2,0)}, \quad \mathcal{D}^{(0,\pm 2)} \tag{2.23}$$

preserves this G-analyticity.

3 Chern-Simons model in $N = 5$ analytic superspace

Using the harmonic-superspace method [12] we introduce the D=3, N=5 analytic matrix gauge prepotentials corresponding to the five harmonic derivatives (2.23)

$$\begin{aligned}
&V^{(p,q)}(\zeta, U) = [V^{(1,1)}, \quad V^{(1,-1)}, \quad V^{(2,0)}, \quad V^{(0,\pm 2)}],\\
&(V^{(1,1)})^\dagger = -V^{(1,-1)}, \quad (V^{(2,0)})^\dagger = V^{(2,0)}, \quad V^{(0,-2)} = [V^{(0,2)}]^\dagger,
\end{aligned} \tag{3.1}$$

where the Hermitian conjugation $\dagger$ includes $\sim$ conjugation of matrix elements and transposition. The infinitesimal gauge transformations of these prepotentials depends on the analytic anti-Hermitian matrix gauge parameter Λ

$$\begin{aligned}
&\delta_\Lambda V^{(1,\pm 1)} = \mathcal{D}^{(1,\pm 1)}\Lambda + [V^{(1,\pm 1)}, \Lambda], \quad \delta_\Lambda V^{(2,0)} = \mathcal{D}^{(2,0)}\Lambda + [V^{(2,0)}, \Lambda],\\
&\delta_\Lambda V^{(0,\pm 2)} = \mathcal{D}^{(0,\pm 2)}\Lambda + [V^{(0,\pm 2)}, \Lambda].
\end{aligned} \tag{3.2}$$

We shall consider the restricted gauge supergroup using the supersymmetry-preserving harmonic (H) analyticity constraints on the gauge superfield parameters

$$H1: \qquad \mathcal{D}^{(0,\pm 2)}\Lambda = 0. \tag{3.3}$$

These constrains yield additional reality conditions for the component gauge parameters.

We use in this paper the harmonic-analyticity constraints on the gauge prepotentials

$$H2: \qquad V^{(0,\pm 2)} = 0, \quad \mathcal{D}^{(0,-2)}V^{(1,1)} = V^{(1,-1)}, \quad \mathcal{D}^{(0,2)}V^{(1,1)} = 0 \tag{3.4}$$

and the conjugated constraints combined with relations (3.1). It is evident that the G- and H-analyticities of the prepotentials are preserved by the restricted gauge transformations (3.3).

Now we have only three gauge prepotentials in complete analogy with the algebraic structure of the gauge theory in the N=3, D=4 harmonic superspace [12]. The superfield CS action can be constructed in terms of these H-constrained gauge superfields

$$
\begin{aligned}
S = -\frac{2i}{3g^2}\int d\mu^{(-4,0)}\mathrm{Tr}\,\{&V^{2,0}\mathcal{D}^{(1,-1)}V^{(1,1)} + V^{1,1}\mathcal{D}^{(2,0)}V^{(1,-1)}\\
&+V^{1,-1}\mathcal{D}^{(1,1)}V^{(2,0)} + V^{2,0}[V^{(1,-1)},V^{(1,1)}] - \tfrac{1}{2}V^{(2,0)}V^{(2,0)}\},
\end{aligned}
\tag{3.5}
$$

where g is the dimensionless CS coupling constant.

The corresponding superfield gauge equations of motion have the following form:

$$
\begin{aligned}
F^{3,-1} &= \mathcal{D}^{(1,-1)}V^{(2,0)} - \mathcal{D}^{(2,0)}V^{(1,-1)} + [V^{(1,-1)},V^{(2,0)}] = 0,\\
F^{3,1} &= \mathcal{D}^{(1,1)}V^{(2,0)} - \mathcal{D}^{(2,0)}V^{(1,1)} + [V^{(1,1)},V^{(2,0)}] = 0, \qquad (3.6)\\
V^{(2,0)} &= \mathcal{D}^{(1,-1)}V^{(1,1)} - \mathcal{D}^{(1,1)}V^{(1,-1)} + [V^{(1.-1)},V^{(1,1)}] \equiv \hat{V}^{(2,0)}. \qquad (3.7)
\end{aligned}
$$

The last prepotential can be composed algebraically in terms of two other basic superfields. Using the substitution $V^{(2,0)} \to \hat{V}^{(2,0)}$ in (3.5) we can obtain the alternative form of the action with only two independent prepotentials $V^{1,1}$ and $V^{1,-1}$

$$
\begin{aligned}
S_2 = -\frac{2i}{3g^2}\int d\mu^{(-4,0)}\mathrm{Tr}\,\{&V^{1,1}\mathcal{D}^{(2,0)}V^{(1,-1)}\\
&+\tfrac{1}{2}\left(\mathcal{D}^{(1,-1)}V^{(1,1)} - \mathcal{D}^{(1,1)}V^{(1,-1)} + [V^{(1,-1)},V^{(1,1)}]\right)^2\}.
\end{aligned}
\tag{3.8}
$$

It is evident that the superfield action (3.5) is invariant with respect to the sixth supersymmetry transformation defined on our gauge prepotentials

$$
\delta_6[V^{(1,\pm1)},V^{(2,0)}] = \epsilon^\alpha_6 D^{(0,0)}_\alpha[V^{(1,\pm1)},V^{(2,0)}], \tag{3.9}
$$

where ϵ^α_6 are the corresponding spinor parameters. Thus, our superfield gauge model possesses the D=3, N=6 supersymmetry.

The D=3, N=5 superconformal transformations can be defined on the analytic coordinates. For instance, the special conformal K-transformations are

$$
\begin{aligned}
\delta_k x^{\alpha\beta}_A &= \tfrac{1}{2}x^{\alpha\gamma}_A\, k_{\gamma\rho}x^{\beta\rho}_A + 2l\, x^{\alpha\beta}_A,\\
\delta_k\theta^{(0,0)\alpha} &= \tfrac{1}{2}x^{\alpha\beta}_A\theta^{(0,0)\gamma}k_{\beta\gamma}, \qquad (3.10)\\
\delta_k\theta^{(1,1)\alpha} &= \tfrac{1}{2}x^{\alpha\beta}_A\theta^{(1,1)\gamma}k_{\beta\gamma} + \tfrac{i}{4}(\theta^{(0,0)})^2\theta^{(1,1)\beta}k^\alpha_\beta, \qquad (3.11)
\end{aligned}
$$

where $k_{\alpha\beta} = k^m(\gamma_m)_{\alpha\beta}$ are the corresponding parameters. The K-transformations of the harmonics have the form

$$
\begin{aligned}
&\delta_k U^{(0,0)}_b = -\lambda^{(1,1)}_k U^{(-1,-1)}_b - \lambda^{(1,-1)}_k U^{(-1,1)}_b, \quad \delta_k U^{(1,1)}_b = \lambda^{(1,1)}_k U^{(0,0)}_b + \lambda^{(2,0)}_k U^{(-1,1)}_b,\\
&\delta_k U^{(1,-1)}_b = \lambda^{(1,-1)}_k U^{(0,0)}_b - \lambda^{(2,0)}_k U^{(-1,-1)}_b, \quad \delta_k U^{(-1,\pm1)}_b = 0, \qquad (3.12)\\
&\lambda^{(1,1)}_k = ik_{\alpha\beta}\theta^{(1,1)\alpha}\theta^{(0,0)\beta}, \quad \lambda^{(1,-1)}_k = ik_{\alpha\beta}\theta^{(1,-1)\alpha}\theta^{(0,0)\beta}, \quad \lambda^{(2,0)}_k = ik_{\alpha\beta}\theta^{(1,1)\alpha}\theta^{(1,-1)\beta}.
\end{aligned}
$$

The special supersymmetry transformations of all coordinates can be obtained via the Lie bracket $\delta_\eta = [\delta_\epsilon, \delta_k]$.

It is easy to check that the analytic integral measure $\mu^{(-4,0)}$ (2.18) is invariant with respect to these superconformal transformations.

The special conformal transformations of the harmonic derivatives have the following form:

$$
\begin{aligned}
&\delta_k \mathcal{D}^{(1,1)} = -\tfrac{1}{2}\lambda_k^{(1,1)}(\mathcal{D}_1^0 + \mathcal{D}_2^0) - \lambda_k^{(1,-1)}\mathcal{D}^{(0,2)},\\
&\delta_k \mathcal{D}^{(1,-1)} = -\tfrac{1}{2}\lambda_k^{(1,-1)}(\mathcal{D}_1^0 - \mathcal{D}_2^0) - \lambda_k^{(1,1)}\mathcal{D}^{(0,-2)},\\
&\delta_k \mathcal{D}^{(2,0)} = \lambda_k^{(2,0)}\mathcal{D}_2^0 + \lambda_k^{(1,-1)}\mathcal{D}^{(1,1)} - \lambda_k^{(1,1)}\mathcal{D}^{(1,-1)},\\
&\delta_k \mathcal{D}^{(0,2)} = \delta_k \mathcal{D}^{(0,-2)} = 0, \quad \delta_k \mathcal{D}_1^0 = \delta_k \mathcal{D}_2^0 = 0,
\end{aligned}
\tag{3.13}
$$

and the SO(5) and special supersymmetry transformations can be defined analogously.

The K-transformations of the gauge prepotentials are

$$
\begin{aligned}
&\delta_k V^{(1,1)} = 0, \quad \delta_k V^{(1,-1)} = 0,\\
&\delta_k V^{(2,0)} = \lambda_k^{(1,-1)} V^{(1,1)} - \lambda_k^{(1,1)} V^{(1,-1)} = \delta_k \hat{V}^{(2,0)},
\end{aligned}
\tag{3.14}
$$

where $\hat{V}^{(2,0)}$ is the composite prepotential (3.7).

It is easy to check directly the superconformal invariance of the gauge actions S (3.5) and S_2 (3.8).

The classical superfield equations (3.6) and (3.7) have only pure gauge solution

$$
V^{(1,\pm 1)} = e^{-\Lambda}\mathcal{D}^{(1,\pm 1)} e^{\Lambda}, \quad V^{(2,0)} = e^{-\Lambda}\mathcal{D}^{(2,0)} e^{\Lambda},
\tag{3.15}
$$

where Λ is an arbitrary anti-Hermitian matrix superfield satisfying the conditions (3.3).

4 Harmonic component fields in the $N = 6$ Chern-Simons model

Let us consider the U(1) gauge group. The pure gauge degrees of freedom in the abelian prepotential $V^{(1,1)}$ can be eliminated by the transformation $\delta V^{(1,1)} = \mathcal{D}^{(1,1)}\Lambda$. In the WZ-gauge we have $\Lambda_{WZ} = ia(x_A)$. The harmonic decomposition of the HA-constrained prepotential $V^{(1,1)}$ in the WZ-gauge has the following form:

$$
\begin{aligned}
&V_{WZ}^{(1,1)} = V_0^{(1,1)} + V_1^{(1,1)} + O(U^2), \quad \mathcal{D}^{(0,-2)} V_{WZ}^{(1,1)} = -[V_{WZ}^{(1,1)}]^{\sim},\\
&V_0^{(1,1)} = (\theta^{(1,1)}\gamma^m\theta^{(0,0)})A_m + i(\theta^{(0,0)})^2\theta^{(1,1)\alpha}\psi_\alpha,
\end{aligned}
\tag{4.1}
$$

$$
\begin{aligned}
&V_1^{(1,1)} = (\theta^{(0,0)})^2 U_a^{(1,1)} B_a + (\theta^{(1,1)}\gamma^m\theta^{(1,-1)})U_a^{(-1,1)} C_m^a\\
&+i\theta^{(1,1)(\alpha}\theta^{(1,-1)\beta}\theta^{(0,0)\gamma)} U_a^{(-1,1)}\Psi^a_{\alpha\beta\gamma} + i(\theta^{(1,-1)}\theta^{(0,0)})\theta^{(1,1)\alpha}U_a^{(-1,1)}\xi^a_\alpha\\
&-i(\theta^{(1,1)}\theta^{(0,0)})\theta^{(1,-1)\alpha}U_a^{(-1,1)}\xi^a_\alpha + i(\theta^{(1,1)})^2(\theta^{(0,0)})^2 U_a^{(-1,-1)} R^a\\
&+i(\theta^{(1,1)}\theta^{(1,-1)})(\theta^{(0,0)})^2 U_a^{(-1,1)} R^a + i(\theta^{(1,1)}\gamma^m\theta^{(1,-1)})(\theta^{(0,0)})^2 U_a^{(-1,1)} G^a_m,
\end{aligned}
\tag{4.2}
$$

where all terms are parametrized by the real off-shell bosonic fields A_m, B_a, C^a_m, R^a and G^a_m or the real Grassmann fields $\psi_\alpha, \Psi^a_{\alpha\beta\gamma}$ and ξ^a_α. The higher harmonic terms in $V_{WZ}^{(1,1)}$ contain an infinite number of the SO(5) tensor fields. In the gauge group SU(n), all component fields are the Hermitian traceless matrices.

Two other U(1) prepotentials contain the same component fields in the WZ-gauge

$$
\begin{aligned}
&V_{WZ}^{(1,-1)} = \mathcal{D}^{(0,-2)} V_{WZ}^{(1,1)} = V_0^{(1,-1)} + V_1^{(1,-1)} + O(U^2),\\
&\hat{V}_{WZ}^{(2,0)} = \mathcal{D}^{(1,-1)} V_{WZ}^{(1,1)} - \mathcal{D}^{(1,-1)} V_{WZ}^{(1,-1)} = V_0^{(2,0)} + V_1^{(2,0)} + O(U^2).
\end{aligned}
\tag{4.3}
$$

The superfield terms

$$\mathrm{Tr}\,[V_0^{(1,1)}\mathcal{D}^{(2,0)}V_0^{(1,-1)}+\tfrac{1}{2}(\mathcal{D}^{(1,-1)}V_0^{(1,1)}-\mathcal{D}^{(1,1)}V_0^{(1,-1)}+[V_0^{(1,-1)},V_0^{(1,1)}])^2]$$

in the action S_2 (3.8) yield the following contribution to the component Lagrangian:

$$L_0=\varepsilon^{mnr}\mathrm{Tr}\,A_m(\partial_nA_r+\tfrac{i}{3}[A_n,A_r])-\tfrac{i}{3}\mathrm{Tr}\,\psi^\alpha\psi_\alpha. \tag{4.4}$$

The superfield terms

$$\begin{aligned}&\mathrm{Tr}\,\{V_1^{(1,1)}\mathcal{D}^{(2,0)}V_1^{(1,-1)}+\tfrac{1}{2}(\mathcal{D}^{(1,-1)}V_1^{(1,1)}-\mathcal{D}^{(1,1)}V_1^{(1,-1)}+[V_0^{(1,-1)},V_1^{(1,1)}]+[V_1^{(1,-1)},V_0^{(1,1)}])^2\}\\&+\mathrm{Tr}\,\{(\mathcal{D}^{(1,-1)}V_0^{(1,1)}-\mathcal{D}^{(1,1)}V_0^{(1,-1)})[V_1^{(1,-1)},V_1^{(1,1)}]\}\end{aligned} \tag{4.5}$$

give us the Lagrangian for the SO(5) vector fields

$$\begin{aligned}L_1=&\tfrac{2}{5}\mathrm{Tr}\,C^a_m(\partial^mB_a+i[A_m,B_a])-\tfrac{8}{15}\mathrm{Tr}\,C^a_mG^m_a-\tfrac{4}{5}\mathrm{Tr}\,B_aR_a\\&+\tfrac{i}{6}\mathrm{Tr}\,\xi^\alpha_a\xi_{a\alpha}-\tfrac{i}{120}\mathrm{Tr}\,\Psi^{a\alpha\beta\gamma}\Psi^a_{\alpha\beta\gamma}.\end{aligned} \tag{4.6}$$

It is not difficult to construct the component Lagrangian for the SO(5) tensor fields.

The N=6 CS equations of motion for the lowest SO(5) component fields are

$$\varepsilon^{mnr}(\partial_nA_r-\partial_rA_n+i[A_n,A_r])=0,\quad \psi_\alpha=C^a_m=B_a=R_a=G^a_m=\xi^a_\alpha=\Psi^a_{\alpha\beta\gamma}=0. \tag{4.7}$$

All SO(5) tensor auxiliary fields also vanish on-shell. The superfield representation of this pure gauge solution in the WZ-gauge is

$$V_{WZ}^{(1,\pm1)}=e^{-ia}\mathcal{D}^{(1,\pm1)}e^{ia},\quad V_{WZ}^{(2,0)}=e^{-ia}\mathcal{D}^{(2,0)}e^{ia}. \tag{4.8}$$

5 Conclusion and acknowledgements

We considered the superfield model with the D=3, N=6 superconformal supersymmetry. The action of this model is constructed in the N=5 harmonic superspace using the Grassmann and harmonic analyticity conditions. The classical superfield equations of motions for the analytic Chern-Simons gauge prepotentials have the pure gauge solution only. In the field-component representation, the action of this model contains the Chern-Simons term for the vector gauge field and an infinite number of the interaction terms for the auxiliary bosonic and fermionic fields. All auxiliary fields vanish on-shell. The superfield representation is useful for the quantization and perturbative calculations.

Note added in proof. P.S. Howe informed me that the harmonic-superspace description of the N=5, 6 Chern-Simons theories was considered in the paper [15]. It should be stressed that our harmonic constraints (3.3) reduce the SO(5)/U(1)×U(1) space to the SO(5)/U(2) space proposed in [15].

I am grateful to E.A. Ivanov for interesting discussions and to P.S. Howe for the important comments. This work was partially supported by DFG grant 436 RUS 113/669-3 , by RFBR grants 06-02-16684 and 06-02-04012, by INTAS grant 05-10000008-7928 and by grant of the Heisenberg-Landau programme.

References

[1] W. Siegel, Nucl. Phys. **B 156** (1979) 135.

[2] J. Schonfeld, Nucl. Phys. **B 185** (1981) 157.

[3] B.M. Zupnik, D.G. Pak, Teor. Mat. Fiz. **77** (1988) 97 [Theor. Math. Phys. **77** (1988) 1070].

[4] B.M. Zupnik, D.G. Pak, Class. Quant. Grav. **6** (1989) 723.

[5] B.M. Zupnik, Phys. Lett. **B 254** (1991) 127; B.M. Zupnik, Teor. Mat. Fiz. **89** (1991) 253 [Theor.Math.Phys. **89** (1991) 1191].

[6] B.M. Zupnik, D.V. Khetselius, Yader. Fiz. **47** (1988) 1147 [Sov. J. Nucl. Phys. **47** (1988) 730].

[7] B.M. Zupnik, Harmonic superspaces for three-dimensional theories, In: Supersymmetries and quantum symmetries, eds. J. Wess and E. Ivanov, Springer Lect. Notes in Phys. **524** (1998) 116; hep-th/9804167.

[8] H.-C. Kao, K. Lee, Phys. Rev. **D 46** (1992) 4691; H.-C. Kao, Phys. Rev. **D 50** (1994) 2881.

[9] E.A. Ivanov, Phys. Lett. **B 268** (1991) 203.

[10] H. Nishino, S.J. Gates, Int. J. Mod. Phys. **8** (1993) 3371.

[11] J.H. Schwarz, JHEP **0411** (2004) 078, hep-th/0411077.

[12] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky, E. Sokatchev, Class. Quant. Grav. 1 (1984) 469;
A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky, E. Sokatchev, Class. Quant. Grav. 2 (1985) 155;
A. Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, Harmonic superspace, Cambridge University Press, Cambridge, 2001.

[13] B.M. Zupnik, Nucl. Phys. **B 554** (1999) 365, Erratum, Nucl. Phys. **B 644** (2002) 405E; hep-th/9902038.

[14] B.M. Zupnik, Gauge theory in D=3, N=5 harmonic superspace, Talk at the International workshop "Supersymmetries and quantum symmetries", JINR, Dubna, 29.07-4.08.2007, arxiv:0708.3951(hep-th).

[15] P.S. Howe, M.I. Leeming, Class. Quant. Grav. **11** (1994) 2843, hep-th/9408062.